Approved Innovative Course

Course: Number Theory
PEIMS Code: N1110025
Abbreviation: NUMTHY
Grade Level(s): 11-12
Number of Credits: 0.5

Course description:

The topics of study contribute to the student's enhanced understanding of historical developments, proofs, and discoveries of mathematical numerical relationships. The study of number theory broadens the student's ability to include not only deductive but inductive reasoning, develop a heightened recognition of numerical relationships, and increase skills in discerning unique mathematical relationships and exercising that intuitive agility in all areas of thought.

Essential knowledge and skills:

(a) Introduction. Student's will broaden their ability to use inductive reasoning and deductive reasoning, develop a heightened recognition of numerical relationships, and increase skills in discerning unique mathematical relationships. This course will be a developmentally appropriate course in applying advanced techniques used by undergraduate majors in mathematics. It will include the following essential concepts in classical Number Theory.

(b) Knowledge and skills.

(1) Introduction to number theory. The student uses essential tools of number theory. The student is expected to:

(A) demonstrate the structure of formal proof which will be used throughout the course;

(B) prove that congruence is an equivalence relation using the concepts of divisibility and congruence;

(C) apply the method of proof by induction to number theory problems;

(D) apply the division algorithm to derive the Euclidean algorithm to compute the greatest common divisor of two numbers; and

(E) determine when a solution to a linear Diophantine equation exists and then categorize all solutions.

(2) Prime numbers. The student investigates the theory of prime numbers including learning about the current state open problems in the field. The student is expected to:

(A) examine unsolved problems in number theory including the Twin Prime Conjecture and the Goldbach Conjecture;

(B) determine the quantity of divisors of a given number from its prime factorization;

(C) determine the sum of divisors of a given number from its prime factorization;

(D) compute the least common multiple of two numbers from the greatest common divisor;

(E) explore Fermat and Mersenne primes;

(F) prove the fundamental theorem of arithmetic;

(G) prove certain roots are irrational using the fundamental theorem of arithmetic ;

(H) develop Euclid's proof of the infinitude of the primes; and

(I) examine arithmetic progressions of primes and Dirichlet's theorem.

(3) Diophantine analysis. The student analyzes linear Diophantine equations. The student is expected to compute the greatest common divisor between two numbers using the Euclidian Algorithm.

(4) Congruence theory. The student derives classical results involving modular arithmetic. The student is expected to:

(A) apply the fundamental definition of congruence under a specified modulus;

(B) define residue systems;

(C) derive the rules for operations on congruences;

(D) show that no polynomial over the rationals will produce only prime outputs;

(E) determine the order of a number under a given modulus;

(F) derive divisibility rules for various primes using congruence theory ;

(G) describe the theory of linear congruences;

(H) solve systems of linear congruences using the Chinese Remainder Theorem;

(I) prove and provide the consequences of Fermat's Little Theorem, Wilson's Theorem and Euler's Theorem;

(J) explain primitive roots;

(K) derive Lagrange's theorem on polynomial solutions modulo a prime;

(L) use Euler-Phi function and prove that it is multiplicative; and

(M) identify Sophie Germane and Germane primes.

Description of specific student needs this course is designed to meet:

This course is an extensive source of information about interesting classical and modern numbers. It will provide problems, facts, and challenges in number lore that may become the beginning of a lifetime adventure with numbers. It will stimulate imaginative solutions to existing problems, to their creative extensions, and to entirely new problems. It is also an essential primer for students who want to ultimately pursue an undergraduate major in mathematics.

Major resources and materials:

Marshall, D., & Odell, E. (2007). Number theory through inquiry. Washington, DC: Mathematical Association of America.

Suggested course activities:

Students will work independently and in groups to debate strategies for proofs and problem solving. Calculators will be used to accelerate the discovery of key concepts. Internet research will also be done on unsolved problems in number theory.

Suggested methods for evaluating student outcomes:

Daily grades will be given on presentations of practice problems using skills and concepts.

Test grades will be given on evaluation of knowledge of concepts and the methods for applying them.

Research projects will be given individually based on research and presentation of the historical developments of number theory. Art work will be evaluated by a committee of two or three teachers.

Teacher qualifications:

Secondary teaching certificate in mathematics,

Recommended: Master’s Degree with at least 15 graduate hours in mathematics, and continuing education in mathematics through graduate courses, summer workshops, and staff development training

Additional information: